Name of College - S.S. College, J. Bad

Deptt - Mathematics

Topic - Problems based on

Plane (Analytical Geometry) of 3-D

B.Sc. I (Hons + Sub)

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Time - 1:00 P.M to 1:45 P.M Date - 06-08-2020

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Problem 1 - O is the origin and A is the point (a, b, c). Find the direction cosines of the joint OA and deduce the equation of the plane through A at right angles to OA.

{ Since direction ratio of st. line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$ }

Direction ratios of OA are $a-0,\ b-0,\ c-0$

i.e. a, b, c

$\therefore$ Direction cosines of OA are

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}},\quad m = \frac{b}{\sqrt{a^2+b^2+c^2}},\quad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

p = Perpendicular distance of plane from origin $= \sqrt{a^2+b^2+c^2}$

Now equation of plane

$$lx + my + nz = p$$

$$\Rightarrow \frac{a}{\sqrt{a^2+b^2+c^2}}x + \frac{b}{\sqrt{a^2+b^2+c^2}}y + \frac{c}{\sqrt{a^2+b^2+c^2}}z = \sqrt{a^2+b^2+c^2}$$

$$\Rightarrow ax + by + cz = a^2+b^2+c^2$$

Problem 2. Find the equation of plane that passes through $(2, -3, 1)$ and is perpendicular to the line joining the Points (3, 4, -1) and (2, -1, 5)

Solution → Let P (3, 4, -1) and Q (2, -1, 5) be two given points. Therefore Direction ratios of PQ be

$2-3, \ -1-4, \ 5-(-1)$ (using Direction ratios $x_2-x_1, y_2-y_1, z_2-z_1$)

$\Rightarrow -1, \ -5, \ 6$

Since plane is perpendicular to PQ therefore, PQ is Normal to the plane.

Thus Equation of plane

$-1x + (-5)y + 6(z) + d = 0$ using Equation of plane $ax+by+cz+d=0$

$\Rightarrow -x - 5y + 6z + d = 0$ —— (1)

Since ~~the~~ point (2, -3, 1) lies On plane therefore, its co-ordinate must satisfies the equation (1)

$-2 - 5(-3) + 6\times 1 + d = 0$

$\Rightarrow -2 + 15 + 6 + d = 0$

$\therefore d = -19$

put $d = -19$ in (1) we have

$-x - 5y + 6z - 19 = 0$

$\Rightarrow x + 5y - 6z + 19 = 0$ it is the required Equation of plane.

Problem 3 → Prove that the Four points (1, 3, -1) (3, 5, 1), (0, 2, -2), (2, 1, -2) are Coplanar and Find the equation of the plane.

Solution:→ Let $A(1,3,-1)$, $B(3,5,1)$, $C(0,2,-2)$ and $D(2,1,-2)$ are given four points. We are to show these points are coplanar.

Say

A	B	C	D
$x_1 = 1$	$x_2 = 3$	$x_3 = 0$	$x_4 = 2$
$y_1 = 3$	$y_2 = 5$	$y_3 = 2$	$y_4 = 1$
$z_1 = -1$	$z_2 = 1$	$z_3 = -2$	$z_4 = -2$

are these points.

We know (x_1,y_1,z_1), (x_2,y_2,z_2), (x_3,y_3,z_3) and (x_4,y_4,z_4) are coplanar if

$$\begin{vmatrix} x_4-x_1 & y_4-y_1 & z_4-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Here $x_2-x_1 = 3-1 = 2$ $\quad x_3-x_1 = 0-1 = -1$ $\quad x_4-x_1 = 2-1 = 1$

$y_2-y_1 = 5-3 = 2$ $\quad y_3-y_1 = 2-3 = -1$ $\quad y_4-y_1 = 1-3 = -2$

$z_2-z_1 = 1-(-1) = 2$ $\quad z_3-z_1 = -2-(-1) = -1$ $\quad z_4-z_1 = -2-(-1) = -1$

$$\text{Now L.H.S} = \begin{vmatrix} 1 & -2 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 2 \\ -1 & -1 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 2 \\ -1 & -1 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 2 \\ -1 & -1 \end{vmatrix}$$

$$= (-2+2) + 2(-2+2) - 1(-2+2)$$

$$= 0 + 2\cdot 0 - 1\cdot 0$$

$$= 0$$

Thus given four points A, B, C, D are coplanar

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for Equation of plane through points $(3,5,1)$, $(0,2,-2)$ and $(2,1,-2)$ is Given by

$$\begin{vmatrix} x-3 & y-5 & z-1 \\ -3 & -3 & -3 \\ -1 & -4 & -3 \end{vmatrix} = 0$$

[using equation of plane
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow (x-3)\begin{vmatrix} -3 & -3 \\ -4 & -3 \end{vmatrix} - (y-5)\begin{vmatrix} -3 & -3 \\ -1 & -3 \end{vmatrix} + (z-1)\begin{vmatrix} -3 & -3 \\ -1 & -4 \end{vmatrix} = 0$$

$$\Rightarrow (x-3)[9-12] - (y-5)[9-3] + (z-1)(12-3) = 0$$

$$\Rightarrow (x-3)(-3) - (y-5)(6) + (z-1)(9) = 0$$
$$\Rightarrow -3x+9-6y+30+9z-9 = 0$$
$$\Rightarrow -3x-6y+9z+30 = 0$$
$$\Rightarrow x+2y-3z-10 = 0$$

Find the equation of plane through the origin and through the intersection of the planes
$$5x-3y+2z+5=0 \quad \text{and} \quad 3x-5y-2z-7=0$$

Solution :→ Since the equation plane through the point of intersection of two planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is Given by

$$(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$$

value of λ is Calculated with the help of

Additional Condition a Conditions Given in problem.

Therefore the equation of plane through the intersection of given planes is

$[5x - 3y + 2z + 5] + \lambda[3x - 5y - 2z - 7] = 0$

$\Rightarrow (5+3\lambda)x - (3+5\lambda)y + (2-2\lambda)z + 5 - 7\lambda = 0$ — ①

Since it passes through $(0,0,0)$
we have $0 - 0 + 0 + 5 - 7\lambda = 0$

$\lambda = \frac{5}{7}$

putting $\lambda = \frac{5}{7}$ in ①

$\left(5 + \frac{3\times5}{7}\right)x - \left(3 + \frac{5\times5}{7}\right)y + \left(2 - \frac{2\times5}{7}\right)z + 5 - \frac{7\times5}{7} = 0$

$\frac{(35+15)}{7}x - \frac{(21+25)}{7}y + \frac{14-10}{7}z + 0 = 0$

$\Rightarrow 50x - 46y + 4z = 0$

$\Rightarrow 25x - 23y + 2z = 0$

Thus the required Equation of Plane is

$25x - 23y + 2z = 0$